Recent Progress on Euler Singularity and Related Models

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Whether the 3D incompressible Euler equations can develop a singularity in finite time from smooth initial data is one of the most challenging problems in mathematical fluid dynamics. This question is closely related to the Clay Millennium Problem on 3D Navier-Stokes Equations. We first review some recent theoretical and computational studies of the 3D Euler equations. We then present strong numerical evidence that the 3D Euler equations develop finite time singularities. A careful local analysis also suggests that the blowing-up solution is highly anisotropic and is not of Leray type. A 1D model is proposed to study the mechanism of the finite time singularity. Very recently we prove rigorously that the 1D model develops finite time singularity. Using a very delicate method of analysis which involves computer assisted proof, we prove the existence of a discrete family of self-similar profiles for a variant of this model. Moreover, we show that the self-similar profile enjoys some stability property.